Towards an Asynchronous, Scalable MLFMA for Three-Dimensional Electromagnetic Problems

B. Michiels* J. Fostier* J. Peeters* I. Bogaert* D. De Zutter*

Abstract —
This paper presents the progress in the development of a scalable parallel MultiLevel Fast Multipole Algorithm (MLFMA) for three-dimensional (3D) electromagnetic problems. Scalability stands for the ability to handle a larger problem on a proportionally larger parallel computer architecture. As a partitioning scheme, hierarchical partitioning (HP) is used, which divides the work load in a very balanced way. This prevents the time, memory and communication complexity per cpu-core from increasing rapidly as a function of the number of cpu-cores and unknowns.

1 INTRODUCTION

To take advantage of the rise of multi-core processors and computational clusters, it is important to develop efficient parallel algorithms. In this paper, 3D electromagnetic simulations are performed using the MLFMA. This algorithm reduces the complexity of a matrix-vector multiplication from $\mathcal{O}(N^2)$ to $\mathcal{O}(N \log N)$, with N the number of unknowns. This allows large problems with a lot of unknowns to be simulated on a single cpu. However, to simulate even larger problems the MLFMA should be partitioned and parallelized as efficiently as possible.

An algorithm is scalable if it can handle a larger problem on a proportionally larger parallel computer architecture with the same parallel efficiency. If the number of cpu-cores $P = \mathcal{O}(N)$, then the time, memory and communication complexity of each cpu-core must be equal to $\mathcal{O}(N/P \log N) = \mathcal{O}(\log N)$, in order to be scalable. The complexity per cpu-core strongly depends on how the problem is partitioned. Three partitioning schemes are discussed in this paper: spatial partitioning (SP), k-space partitioning (KP), and hierarchical partitioning (HP).

2 PARTITIONING SCHEMES

The MLFMA creates a tree-like structure of boxes and in all the boxes of every MLFMA-level a radiation pattern is calculated. On every level, $\mathcal{O}(N)$ computations must be performed and the number of levels is $\mathcal{O}(\log N)$.

*Department of Information Technology (INTEC), Ghent University, Ghent, B-9000, Belgium http://emweb.intec.ugent.be

In the spatial partitioning scheme, each box with its radiation pattern is assigned to a certain cpu-core. For the bottom levels of the MLFMA-tree, there are $\mathcal{O}(N)$ boxes and the size of their radiation patterns is $\mathcal{O}(1)$. In this case the boxes can be distributed among the $\mathcal{O}(N)$ cpu-cores. However, at the top levels, the number of boxes is $\mathcal{O}(1)$ and their radiation patterns have $\mathcal{O}(N)$ size. The cpu-cores that contain the boxes of the top levels will exceed the maximal complexity of $\mathcal{O}(\log N)$, so SP is not scalable.

The idea of k-space partitioning is to share the radiation pattern of an MLFMA-box among the different cpu-cores. This divides the work load on the top levels in a balanced way among all the cpu-cores. However, at the bottom levels, the size of the radiation pattern is $\mathcal{O}(1)$ and cannot be distributed over $\mathcal{O}(N)$ cpu-cores. Therefore also KP is not scalable.

A gradual transition from spatial to k-space partitioning can lead to a scalable partitioning. This scheme is called hierarchical partitioning [1] and in 2D its scalability has been proven [2, 3].

3 HIERARCHICAL PARTITIONING IN 3D

In 3D, for high-frequency problems, the number of boxes decreases by a factor 4, while the size of the radiation pattern increases by a factor 4, when one goes one level up in the MLFMA-tree. Therefore, in our implementation, a radiation pattern is partitioned in 4^i parts.

Fig. 1 shows the hierarchical partitioning for 3 MLFMA-levels among 16 cpu-cores. One should be careful how to partition the radiation pattern. A "strip-wise"-partitioning, i.e. a partitioning in which only the range of, for example, the ϕ-values is partitioned, results in a communication complexity of $\mathcal{O}(\sqrt{N})$ per cpu-core per level [2]. Hence, "block-wise"-partitioning, subdividing both the range of ϕ- and θ-values, must be adopted as this only requires $\mathcal{O}(1)$ communication per cpu-core per level.

4 RESULTS

To check the scalability of the HP we consider the electromagnetic problem of a plane wave that scat-

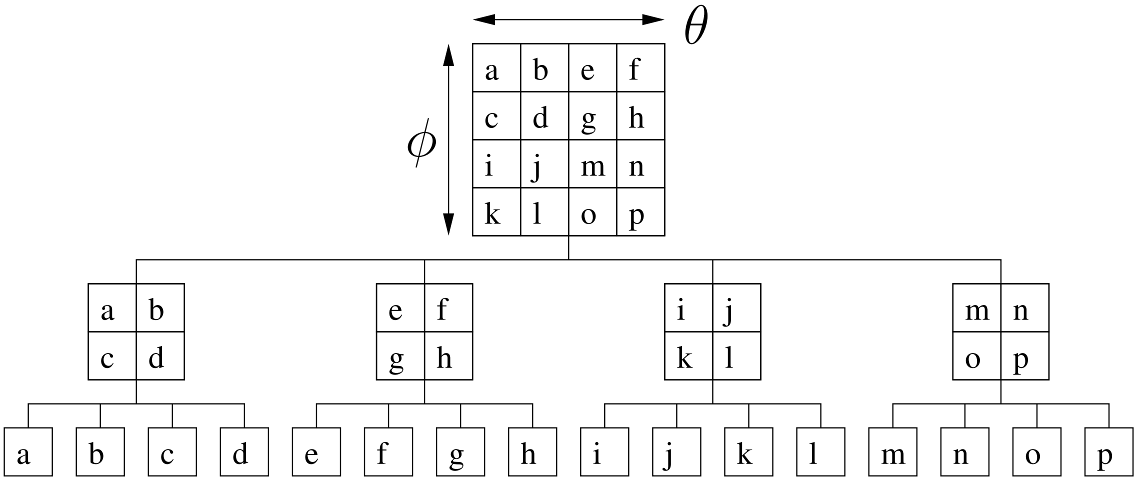

Figure 1: Hierarchical partitioning in 3D: the boxes denote the radiation patterns, the letters denote the cpu-cores.

ters at a Perfectly Electrically Conducting (PEC) cuboid. We start from a simulation with 4 cpu-cores, 18432 unknowns and 4 MLFMA-levels and with each step we increase the number of unknowns N and the number of cpu-cores P by a factor 4. For $P = 1024$ the number of unknowns is 4 718 592 and the MLFMA-tree contains 8 levels. Fig. 2 displays the total communication of the cpu-core with the largest amount of communication (normalized by $1/\log(P)$) as a function of the number of cpu-cores P and unknowns N.

As one can see from Fig. 2 the communication complexity of each cpu-core does not exceed $\mathcal{O}(\log N)$ for HP. This is much better dan SP, where the communication complexity, scaled by $1/\log(P)$, grows as a function of P and N, and, as a consequence, is higher than $\mathcal{O}(\log N)$. Fig. 2 shows that HP is a scalable partitioning scheme, while SP is not.

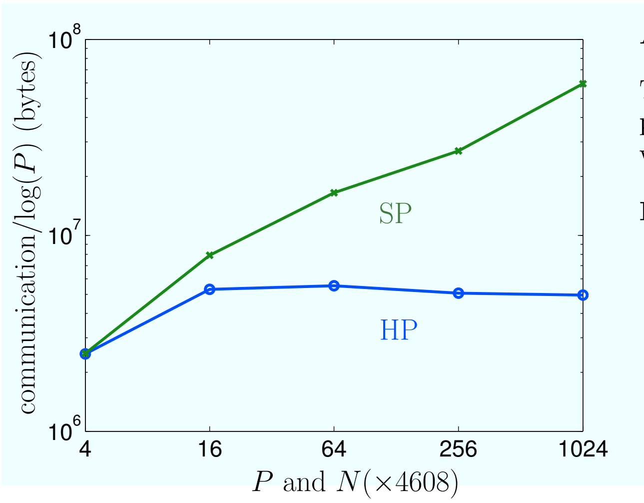

Figure 2: Maximal communication per cpu-core scaled by $1/\log(P)$ as a function of the number of cpu-cores P and unknowns N.

5 CONCLUSION

In this paper a scalable MLFMA in 3D is implemented, using a hierarchical partitioning scheme. In order to be scalable, the complexity of every cpu-core should not be higher than $\mathcal{O}(\log N)$. This paper shows that the communication complexity of each cpu-core remains below the maximal complexity of $\mathcal{O}(\log N)$, which implies that the implementation is scalable. The results are compared with a spatial partitioning scheme, where the communication complexity did exceed $\mathcal{O}(\log N)$.

Acknowledgment

The computational resources (Stevin Supercomputer Infrastructure) and services used in this work were provided by Ghent University.

References

[1] Ö. Ergül and L. Gürel, "Hierarchical Parallelisation Strategy for Multilevel Fast Multipole Algorithm in Computational Electromagnetics", Electronics Letters, 44(1):3–5, 2008

[2] J. Fostier and F. Olyslager, "Provably Scalable Parallel Multilevel Fast Multipole Algorithm", Electronics Letters 44(19):1111–1112, 2008.

[3] J. Fostier and F. Olyslager, "Full-Wave Electromagnetic Scattering at Extremely Large 2-D Objects", Electronics Letters 45(5), 2009.